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7/12/18

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **88**

(3)

Unique Paper Code : **32351101** **I**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

(Attempt any *four* questions from Section I)

1. If $y = \log (x+\sqrt{x^2+1})$, show that :

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

2. Sketch the graph of the function

$$f(x) = \frac{3x-5}{x-2}$$

by determining all critical points, intervals of increase and decrease, points of relative maxima and minima, concavity of the graph, inflection points and horizontal and vertical asymptotes.

P.T.O.

3. Evaluate : $\lim_{x \to 0} (e^x - 1 - x)^x$.

4. Given the cost $C(x) = \frac{1}{8}x^2 + 5x + 98$ of producing x units of a particular commodity and the selling price $p(x) = \frac{1}{2}(75 - x)$ when x units are produced. Determine the level of production that maximizes profit.

5. Sketch the graph of $r = \sin 2\theta$ in polar coordinates.

Section II

(Attempt any *four* questions from Section II)

6. Obtain the reduction formula for

$$\int \sec^n x \, dx.$$

Use it to evaluate $\int \sec^6 x \, dx$.

7. Find the volume of the solid generated by revolving the region enclosed by $y = x$, $y = 2 - x^2$ and $x = 0$ is revolved about the x-axis.

8. Use cylindrical shells method to find the volume of the solid generated when the region enclosed by $y = 2x - x^2$ and $y = 0$ is resolved about y-axis.

9 Show that the arc length of the curve $y = \cosh x$ between $x = 0$ and $x = \log 2$ is 3/4.

10. Find the area of the surface generated by revolving the curve $y = \sqrt{9 - x^2}$, $-1 \le x \le 1$, about x-axis.

Section III

(Attempt any *three* questions from Section III)

11. Find the equation of parabola having axis $y = 0$ and passing through the points (3, 2) and (2, –3).

12. Find the equation of ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.

13. Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

14. Rotate the coordinate axes to remove the xy-term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Identify the resultant conic.

Section IV

(Attempt any *four* questions from Section IV)

15. Given the vector functions

$$\vec{\mathbf{F}}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and
$$\vec{\mathbf{G}}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t\to 1}[\vec{\mathbf{F}}(t)\times\vec{\mathbf{G}}(t)] = [\lim_{t\to 1}\vec{\mathbf{F}}(t)] \times [\lim_{t\to 1}\vec{\mathbf{G}}(t)].$$

16. A velocity of particle moving in space is

$$\vec{\mathbf{V}}(t) = t^2\hat{\mathbf{i}} - e^{2t}\hat{\mathbf{j}} + \sqrt{t}\hat{\mathbf{k}} \cdot$$

Find the particle's position as a function of t if the position at time $t = 0$ is $\vec{\mathbf{R}}(0) = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

17. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from ground level :

(*i*) Find the maximum height attained by the shell.

(*ii*) Find the time of flight and the range of the shell.

18. Find the tangential and normal components of the acceleration of an object that moves with position vector

$$\vec{\mathbf{R}}(t) = \cos t\hat{\mathbf{i}} + \sin t\hat{\mathbf{j}} + t\hat{\mathbf{k}}.$$

19. Find the curvature $\kappa(t)$ for the curve given by the vector equation

$$\vec{\mathbf{R}}(t) = 4\cos t\hat{\mathbf{i}} + 4\sin t\hat{\mathbf{j}} + t\hat{\mathbf{k}}\ (0 \le t \le 2\pi).$$

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S. No. of Question Paper : **89** (4)

Unique Paper Code : **32351102** **I**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Find polar representation of the complex number : 6

$$z_{*} = \sin a + i(1 + \cos a),\ a \in [0, 2\pi).$$

(*b*) Find $|z|$ and $\arg z$, $\arg(-z)$ for : 6

(*i*) $z = (1 - i)(6 + 6i)$

(*ii*) $z = (7 - 7\sqrt{3}i)(-1 - i)$.

(*c*) Solve the equation : 6

$$z^4 = 5(z - 1)(z^2 - z + 1).$$

2. (*a*) For $a, b \in \mathbf{Z}$, define $a \sim b$ iff $a^2 - b^2$ is divisible by 3 : 6

(*i*) Prove that $\sim$ is an equivalence relation on Z.

(*ii*) Find the equivalence classes of 0 and 1.

(*b*) Define : 6

$$f : \mathbf{Z} \to \mathbf{Z} \text{ by } f(x) = x^2 - 5x + 5$$

(*i*) Is f one-to-one ?

(*ii*) Is f onto ?

Justify each answer.

P.T.O.

(c) Show that the open intervals (0, 1) and (4, 6) have the same cardinality. 6

3. (a) Suppose a, b and c are three non-zero integers with a and c relatively prime. Show that : 6

$$\gcd(a, bc) = \gcd(a, b).$$

(b) (i) Solve the following congruence if possible. If no solution exists, explain why not :

$$4x \equiv 2 \pmod 6.$$

(ii) Find three positive and three negative integers in $\overline{5}$ w.r.t. congruence mod 7. 6

(c) Use mathematical induction to establish the following inequality : 6

$$n\,! > n^3, \text{ for all } n \geq 6.$$

4. (a) Find the general solution to the following linear system : 6½

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$
$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15.$$

(b) Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$.

Is u in the subspace of $\mathbf{R}^3$ spanned by the columns of A. Why or why not ? 6½

(c) Let :

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

(i) For what values of h is v_3 in span $\{v_1, v_2\}$?

(ii) For what values of h is $\{v_1, v_2, v_3\}$ linearly dependent ? Justify each answer. 6½

5. (a) Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$, and define by $T : R^3 \to R^3$ by $T(x) = Ax$. Find all x in R^3 such that $T(x) = 0$. Does $b = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ belong to range of T ? 6½

(b) A linear transformation $T : R^2 \to R^2$ first reflects points through the x_1-axis and then reflects points through the x_2-axis. Show that T can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation ? 6½

(c) Let

$$A = \begin{bmatrix} 2 & -3 & -4 \\ -8 & 8 & 6 \\ 6 & -7 & -7 \end{bmatrix} \text{ and } u = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}.$$

Is u in Nul A ? Is u in Col A ? Justify each answer. 6½

6. (*a*) Given $b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and $B = \{b_1, b_2\}$ is basis of subspace H of R^2.

(*i*) Determine if $x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ belongs to H.

(*ii*) Find $[x]_B$, the B-coordinate vector of x. 6½

(*b*) Determine the basis of the null space of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix}.$$ 6½

(*c*) Is $\lambda = -2$ an eigenvalue of $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

If so, find one corresponding eigenvector. 6½

Sl. No. 07 Q.P. 1686 2018

I

Unique Paper Code :235101

Name of the Course :B.Sc.(Hons.) Mathematics

Name of the Paper :Calculus I/ Paper Code: MAHT 101 (5)

Semester :I

Duration :3 hours Maximum Marks: 75

Instructions for Candidates

- *Write your Roll No. on the top immediately on receipt of this question paper.*
- All the Sections are compulsory
- Use of non-programmable scientific calculator is allowed.
- All questions carry equal marks.

SECTION – I

Attempt any **four** questions from Section – I

1. If

$$y = e^{(m\cos^{-1}x)},$$

Prove that :

$$(1-x^2)\,y_{n+2} - (2n+1)\,x\,y_{n+1} - (n^2+m^2)y_n = 0.$$

2. Find all value of A and B so that

$$\lim_{x\to 0}\frac{\sin Ax + Bx}{x^3} = 36.$$

3. Find constants a, b and c that guarantee that the graph of

$$f(x) = ax^3 + bx^2 + c$$

will

Will have a relative extremum at $(2, 11)$ and an inflection point at $(1,5)$.

4. A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in rupees) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200.$$

and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit. Determine the price that corresponds to the maximum profit.

5. Compute :

$$\int_0^{\frac{\pi}{2}} Sin^2\frac{x}{2}\, Cos^2\frac{x}{2}\, dx.$$

SECTION–II

Attempt any four questions from Section – II

6. Trace the polar curve

$$r = 1 + 2\sin\theta.$$

7. Find the volume of the solid that results when the region enclosed by the given curves $y = x^2$, $x = 2$ and $x = 0, y = 0$ is revolved about x -axis.

8. Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y= \sqrt{x}$, $x =1$ and $x=4$ and x-axis is revolved about y-axis.

9. Find the arc length of the curve $x = \frac{1}{3}t^3$, $y = \frac{1}{2}t^2$, $(0 \le t \le 1)$

10. Find the area of the surface generated by revolving $x = 9y+1$, $0 \le y \le 2$ about y axis.

SECTION –III

Attempt any **three** questions from Section – III

11. Find an equation of the ellipse traced by a point which moves so that the sum of its distance to (4, 1) and (4, 5) is 12.

12. Describe the graph of the curve

$$y = 4x^2 + 8x + 5$$

13. Trace the conic

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

by rotating the coordinate axes to remove the xy- term.

14. Find a polar equation for the ellipse that has its focus at the pole, directrix above the pole, c = 5 and e = 1/5.

SECTION –IV

Attempt any **four** questions from Section – IV

15. Find $\lim_{t \to 2} F(t)$,

where $F(t) = [(2t\,\mathbf{i} - 5\,\mathbf{j} + e^t\mathbf{k}) \times (t^2\mathbf{i} + 4\sin t\,\mathbf{j})]$.

16. Find the tangent vector and parametric equation of tangent line to the graph of the vector function

$$F(t) = t^2\mathbf{i} + (\cos t)\,\mathbf{j} + (t^2\cos t)\,\mathbf{k} \quad \text{at } t = \pi/2.$$

17. State and prove Kepler's Second Law.

18. A baseball hit at 24^0 angle from 3 ft. above the ground just goes over the 9 ft. fence 400 ft. from home plate .About how fast was the ball travelling and how long did it take the ball to reach the wall?

19. An object moves along the curve $= 5(1 + \cos\theta), \theta = 2t + 1$. Find its velocity and acceleration in terms of unit polar vectors u_r and u_θ.

Your Roll No. 2018

[This question paper contains 4 printed pages]

Sl. No. of Q.P. 1688

I

Unique Paper Code: 235104

Name of the Paper: MAHT 103- Algebra I

Name of the Course: B.Sc. (Hons.) Mathematics

Semester: I

Duration: 3 Hours

Maximum Marks: 75

(6)

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper
2. Do any two parts from each question.

1. (a) Find the polar representation of the complex number

$$z = 1 + \cos a + i \sin a,\ a \in (0, 2\pi)$$ **(6.5)**

(b) Prove that $\sin 5t = 16\sin^5 t - 20\sin^3 t + 5$ **(6.5)**

(c) Solve the equation $z^3 - 125 = 0$. **(6.5)**

2. (a) Let $M_i(z_i)$, $i \in \{1, 2, 3, 4\}$ be four distinct points. Prove that

(i) The points M_1, M_2, M_3 are collinear if and only if

$$\frac{z_3 - z_1}{z_2 - z_1} \in R^*$$

(ii) The lines M_1M_2 and M_3M_4 are orthogonal if and only if

$$\frac{z_1 - z_2}{z_3 - z_4} \in iR^*$$ (6)

(b) Find the roots of

$$x^4 - x^3 - 7x^2 + 23x - 20 = 0$$

given that the product of the two roots is 5. (6)

(c) Using Descartes' rule of signs, show that the equation

$$x^{11} + x^8 - 3x^5 + x^4 + x^3 - 2x^2 + x - 2 = 0$$

must have at least four complex roots. (6)

3. (a) For a, b ∈ **R**, define a ~ b if and only if $a - b \in \mathbf{Z}$.

(i) Prove that ~ defines an equivalence relation on **Z**.

(ii) Find the equivalence class of 5? (5)

(b) Give an example of a relation which is

(i) reflexive but not symmetric.

(ii) transitive but not reflexive. (5)

Bold

(c) Define $f : Z \to Z$ by $f(x) = 3x^3 - x$.

(i) Is f one-to-one?

(ii) Is f onto? (5)

4. (a) Suppose $A = \{x \in R : x \le 0\}$, $B = \{x \in R : x \ge 0\}$ and define $f : A \to B$ by $f(x) = x^2$. Find inverse of f. (5)

(b) Show that the sets $(2, 5)$ and $(10, \infty)$ have same cardinality. (5)

(c) If p is a prime that divides ab, prove that p divides a or p divides b. (5)

5. (a) (i) Find the Standard matrix A for the dilation transformation $T(x) = 3x$.

(ii) Show that the transformation T defined by

$T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear. (7.5)

(b) (i) Define linearly dependent and linearly independent set of vectors?

(ii) Determine if the columns of the matrix form a linearly independent set.

$$A = \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$$ (7.5)

(c) By applying elementary row operations find the inverse, if it exists, of the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$ **(7.5)**

6. (a) Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one to one transformation. Does T map R^2 onto R^3? **(7.5)**

(b) Find the basis for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$ **(7.5)**

(c) Find the characteristic equation of

$$A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$$ (7.5)

Also find the eigen values.

Sl. No. of Q.P : 1687 2018

Unique Paper Code : **235103**

Name of the Paper : **Analysis- I**

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : **I** (7)

Duration : 3 Hours **Maximum Marks : 75**

(Write your Roll No. On the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

1 (a) Define supremum of a non-empty subset of **R**. Prove that a number u is the supremum of a non-empty subset S of **R** if and only if

(i) $s \leq u \quad \forall s \in S$;

(ii) for any $\varepsilon > 0$, there exists $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$. 5

(b) What is the supremum of the set $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbf{N}\right\}$. Justify your answer. ? 5

(c) Prove that between any two real numbers $x,\ y$ with $x < y$ there exists a rational number r such that $x < r < y$. 5

2 (a) Let S be a non- empty subset of **R** and is bounded below. Prove that

$$-\sup\{-s : s \in S\} = \inf S.$$ 5

(b) Show that a finite intersection of open sets is an open set and hence deduce that finite union of closed sets is closed. 5

(c) Define limit point of a set. Find limit points of $[0,1]$. 5

3 (a) (i) Show that every convergent sequence is bounded. Is the converse true? Justify your answer. 5

(ii) Let (x_n) be a sequence of real numbers that converges to x. Show that the sequence $(|x_n|)$ converges to $|x|$. 2½

(b) (i) Let (x_n) be a sequence of positive real numbers such that $L = \lim_n \left(\frac{x_{n+1}}{x_n}\right)$ exists. If $L < 1$, then (x_n) converges and $\lim_n x_n = 0$. Show that 5

(ii) Determine $\lim_n \left(\frac{n}{b^n}\right)$ for $b > 1$. 2½

(c) (i) State and prove Monotone Convergence Theorem. 5

(ii) List the first seven terms of the following inductively defined sequence:

$$x_1 = 3,\ x_2 = 5,\ x_{n+2} = x_n + x_{n+1}\,.$$ 2½

4 (a) State and prove Cauchy Convergence criterion for sequence of real numbers. 5

(b) Let $x_n = \sqrt{n+1} - \sqrt{n}$ for $n \in \mathbf{N}$. Show that the sequences (x_n) and $(\sqrt{n}x_n)$ converge. Find their limits. 5

(c) Show that the sequence $\left(\frac{n+1}{n}\right)$ is Cauchy but $((-1)^n)$ is not a Cauchy sequence. 5

5 (a) Find the limit inferior and limit superior of the following sequences:

(i) $((-1)^n)$ (ii) $(1, 1, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$ (iii) $(\sin\frac{n\pi}{3})$. 5

(b) State and prove Bolzano Weierstrass Theorem for sequences.

(c) Let $x_1 = 1$ and $x_{n+1} = \sqrt{x_n + 2}$, $n \in \mathbf{N}$. Show that (x_n) converges and find its limit. 5

6 (a) Give examples of the following series with justifications.

(i) a divergent series $\sum a_n$ for which $\sum a_n^2$ converges.

(ii) a convergent series $\sum a_n$ for which $\sum a_n^2$ diverges. 5

(b) Test the convergence of any two of the following series:

(i) $\sum \frac{n-1}{n^2}$

(ii) $\sum \frac{1}{2^n + n}$

(iii) $\sum \frac{n^2}{n!}$ 5

(c) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of positive terms. If the sequence $\left(\frac{a_n}{b_n}\right)$

converges with a positive limit, then prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if

$\sum_{n=1}^{\infty} b_n$ converges. 5

7 (a) Check for convergence and absolute convergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

(ii) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}-\sqrt{n}}{n}$ 5

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$. 5

(c) Examine the convergence of any two of the following series:

(i) $\sum_{n=1}^{\infty} \frac{n2^n}{n^2+1}$

(ii) $\sum_{n=1}^{\infty} \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^{1/2}$

(iii) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ 5